Math 101 Fall 2002 Exam 2

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Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have **one hour and fifteen minutes**. Do all 8 problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. An answer with no supporting work will receive no credit.
Please print you name clearly here.

Print name: _____

Upon finishing please sign the pledge below:
On my honor I have neither given nor received any aid on this exam.

_____

Grader's use only:

1. _____ /15

2. _____ /10

3. _____ /10

4. _____ /20

5. _____ /5

6. _____ /5

7. _____ /5

8. _____ /20

1. [15 points] Evaluate the following limits, if they exist.

   (a) $\lim_{x \to -3} \frac{x^2+x-6}{x^3-8x+3}$

   (b) $\lim_{t \to 0} \frac{t-\tan t}{t^3}$

   (c) $\lim_{x \to 0} (1+3x)^{-1/x}$

2. [10 points] Find the first three derivatives of the function $f(x) = e^{\cos x}$.

3. [10 points] A spherical balloon is being inflated at the rate of 32π cm^3/sec. When the radius is 4cm, at what rate is the radius increasing?

4. [20 points] Evaluate the following indefinite integrals:

   a. $\int \frac{3}{(2x+1)^3} dx$

   b. $\int 2\cos(3x)(\sin(3x))^4 dx$

   c. $\int \sqrt{x}(1+x^{3/2})^{1/3} dx$

Find the value of the following definite integrals:

   d. $\int_1^2 \frac{1+(1/t)}{t^2} dt$

   e. $\int_0^{\ln 3} \frac{e^x}{1+2e^x} dx$

5. [5 points] Solve $\frac{dy}{dx} = (x-1)^9, y(0) = 0$

6. [5 points] Differentiate the following function $f(x) = \int_2^{\sin(x)} (\ln t)^2 dt$

7. [5 points] Express $\sum_{i=1}^{n} (i+2)(i-1)^2$ as a polynomial in n.

8. [20 points] For the function $f(x) = (x^2+7)e^{-x/4}$, the first two derivatives are $f'(x) = -\frac{1}{4}(x-1)(x-7)e^{-x/4}$ and $f''(x) = \frac{1}{16}(x-3)(x-13)e^{-x/4}$.

(a) Find (and justify) all horizontal and vertical asymptotes of the graph $y = f(x)$. At the vertical asymptotes compute both the left and right hand limits of $f(x)$.

(b) Find the intervals on which $f(x)$ is increasing and those on which it is decreasing.

(c) Find the critical points of $f(x)$ and classify them as local maxima, local minima or neither.

(d) Find the intervals on which $f(x)$ is concave upward and those on which it is concave downward.

(e) Sketch the graph of $y = (x^2 + 7)e^{-x/4}$ showing the results of (a)-(d). (The following values may be helpful $f(1) \approx 6.23$, $f(3) \approx 7.56$, $f(7) \approx 9.73$, $f(13) \approx 6.82$.)