Math 101 Fall 2002 Exam 1

Instructor: Richard Evans/Richard Stong

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Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have **one hour and fifteen minutes**. Do all 7 problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. An answer with no supporting work will receive no credit.
Please print you name clearly here.

Print name: _____

Upon finishing please sign the pledge below:
On my honor I have neither given nor received any aid on this exam.

_____

Grader's use only:

1. _____ /15

2. _____ /15

3. _____ /10

4. _____ /20

5. _____ /10

6. _____ /15

7. _____ /15

1. [15 points] Evaluate the following limits, if they exist.

   (a) $\lim_{x \to -2} \frac{x^2+x-2}{x^2-x-6}$

   (b) $\lim_{\theta \to 0} \frac{\tan 2\theta}{\sin 5\theta}$

   (c) $\lim_{x \to 1} \frac{\frac{2}{x+1}-1}{x-1}$

2. [15 points] Suppose c is a constant and the function f is given by:

$$f(x) = \begin{cases} c^2 - x^2, & x < 0 \\ 2(c-x)^2, & x \geq 0 \end{cases}$$

(a) Calculate the following limits:

$$\lim_{x \to 0^-} f(x) \qquad \text{and} \qquad \lim_{x \to 0^+} f(x)$$

(b) Find a value of the constant c so that the function f is continuous everywhere.

3. [10 points] Find the derivative of $f(x) = \sqrt{x+3}$ **using the definition of the derivative**. (No credit will be given for finding the derivative by other means.)

4. [20 points] Calculate the derivative for each of the following functions:

(a) $(4x^2 + 7x + 3)^{50}$

(b) $(1 + 2x)^5 \sin(2x^3)$

(c) $3 + \frac{2x}{\sqrt{x+1}}$

(d) $\cos^2(3e^x)$

5. [10 points] Find the equation of the tangent line to the graph of $y = \tan(2x) + 3\sec x$ at the point $(0, 3)$.

6. [15 points] Find the maximum and minimum value of $f(x) = \frac{1-x}{x^2+3}$ on $[-2, 1]$. Be sure to show all the steps you need to show in order to justify that your answers really are the maximum and minimum.

7. [15 points] A rectangle of perimeter 24 inches is rotated about one of its sides to generate a right circular cylinder. What are the dimensions of the rectangle which give a cylinder of maximal volume? (Recall that the volume of a right circular cylinder with height h and radius r is $V = \pi r^2 h$.)